NAG Toolbox for MATLAB

e02aj

1 Purpose

e02aj determines the coefficients in the Chebyshev-series representation of the indefinite integral of a polynomial given in Chebyshev-series form.

2 Syntax

```
[aint, ifail] = e02aj(n, xmin, xmax, a, ia1, qatm1, iaint1)
```

3 Description

e02aj forms the polynomial which is the indefinite integral of a given polynomial. Both the original polynomial and its integral are represented in Chebyshev-series form. If supplied with the coefficients a_i, for $i = 0, 1, \ldots, n$, of a polynomial $p(x)$ of degree n, where

$$p(x) = \tfrac{1}{2}a_0 + a_1 T_1(\bar{x}) + \cdots + a_n T_n(\bar{x}),$$

the function returns the coefficients a'_i, for $i = 0, 1, \ldots, n+1$, of the polynomial $q(x)$ of degree $n+1$, where

$$q(x) = \tfrac{1}{2}a'_0 + a'_1 T_1(\bar{x}) + \cdots + a'_{n+1} T_{n+1}(\bar{x}),$$

and

$$q(x) = \int p(x)dx.$$

Here $T_j(\bar{x})$ denotes the Chebyshev polynomial of the first kind of degree j with argument $\bar{x}$. It is assumed that the normalized variable $\bar{x}$ in the interval $[-1, +1]$ was obtained from your original variable x in the interval $[x_{\min}, x_{\max}]$ by the linear transformation

$$\bar{x} = \frac{2x - (x_{\max} + x_{\min})}{x_{\max} - x_{\min}}$$

and that you require the integral to be with respect to the variable x. If the integral with respect to $\bar{x}$ is required, set $x_{\max} = 1$ and $x_{\min} = -1$.

Values of the integral can subsequently be computed, from the coefficients obtained, by using e02ak.

The method employed is that of Chebyshev-series (see Chapter 8 of Modern Computing Methods 1961), modified for integrating with respect to x. Initially taking $a_{n+1} = a_{n+2} = 0$, the function forms successively

$$a'_i = \frac{a_{i-1} - a_{i+1}}{2i} \times \frac{x_{\max} - x_{\min}}{2}, \qquad i = n+1, n, \ldots, 1.$$

The constant coefficient a'_0 is chosen so that $q(x)$ is equal to a specified value, **qatm1**, at the lower end point of the interval on which it is defined, i.e., $\bar{x} = -1$, which corresponds to $x = x_{\min}$.

4 References

Modern Computing Methods 1961 Chebyshev-series *NPL Notes on Applied Science* **16** (2nd Edition) HMSO

5 Parameters

5.1 Compulsory Input Parameters

1: **n – int32 scalar**

n, the degree of the given polynomial $p(x)$.

Constraint: $\mathbf{n} \geq 0$.

2: **xmin – double scalar**

3: **xmax – double scalar**

The lower and upper end points respectively of the interval $[x_{\min}, x_{\max}]$. The Chebyshev-series representation is in terms of the normalized variable $\bar{x}$, where

$$\bar{x} = \frac{2x - (x_{\max} + x_{\min})}{x_{\max} - x_{\min}}.$$

Constraint: **xmax** > **xmin**.

4: **a(la) – double array**

The Chebyshev coefficients of the polynomial $p(x)$. Specifically, element $i \times \mathbf{ia1} + 1$ of **a** must contain the coefficient a_i, for $i = 0, 1, \ldots, n$. Only these $n + 1$ elements will be accessed.

Unchanged on exit, but see **aint**, below.

5: **ia1 – int32 scalar**

The index increment of **a**. Most frequently the Chebyshev coefficients are stored in adjacent elements of **a**, and **ia1** must be set to 1. However, if for example, they are stored in **a**(1), **a**(4), **a**(7), ..., then the value of **ia1** must be 3. See also Section 8.

Constraint: $\mathbf{ia1} \geq 1$.

6: **qatm1 – double scalar**

The value that the integrated polynomial is required to have at the lower end point of its interval of definition, i.e., at $\bar{x} = -1$ which corresponds to $x = x_{\min}$. Thus, **qatm1** is a constant of integration and will normally be set to zero by you.

7: **iaint1 – int32 scalar**

The index increment of **aint**. Most frequently the Chebyshev coefficients are required in adjacent elements of **aint**, and **iaint1** must be set to 1. However, if, for example, they are to be stored in **aint**(1), **aint**(4), **aint**(7), ..., then the value of **iaint1** must be 3. See also Section 8.

Constraint: $\mathbf{iaint1} \geq 1$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

np1, la, laint

5.4 Output Parameters

1: **aint(laint) – double array**

The Chebyshev coefficients of the integral $q(x)$. (The integration is with respect to the variable x, and the constant coefficient is chosen so that $q(x_{\min})$ equals **qatm1**). Specifically, element $i \times \mathbf{iaint1} + 1$ of **aint** contains the coefficient a'_i, for $i = 0, 1, \ldots, n + 1$. A call of the function may have the array name **aint** the same as **a**, provided that note is taken of the order in which elements are overwritten when choosing starting elements and increments **ia1** and **iaint1**: i.e., the coefficients, $a_0, a_1, \ldots, a_{i-2}$ must be intact after coefficient a_i' is stored. In particular it is possible to overwrite the a_i entirely by having **ia1** = **iaint1**, and the actual array for **a** and **aint** identical.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, **np1** < 1,
or **xmax** ≤ **xmin**,
or **ia1** < 1,
or **la** ≤ (**np1** − 1) × **ia1**,
or **iaint1** < 1,
or **laint** ≤ **np1** × **iaint1**.

7 Accuracy

In general there is a gain in precision in numerical integration, in this case associated with the division by $2i$ in the formula quoted in Section 3.

8 Further Comments

The time taken is approximately proportional to $n + 1$.

The increments **ia1**, **iaint1** are included as parameters to give a degree of flexibility which, for example, allows a polynomial in two variables to be integrated with respect to either variable without rearranging the coefficients.

9 Example

```
n = int32(6);
xmin = -0.5;
xmax = 2.5;
a = [2.53213;
     1.13032;
     0.2715;
     0.04434;
     0.00547;
     0.00054;
     4e-05];
ia1 = int32(1);
qatm1 = 0;
iaint1 = int32(1);
[aint, ifail] = e02aj(n, xmin, xmax, a, ia1, qatm1, iaint1)
```

```
aint =
    2.6946
    1.6955
    0.4072
    0.0665
    0.0082
    0.0008
    0.0001
    0.0000
```

```
ifail =

           0
```